Design Study on a Superconducting Multicell RF Accelerating Cavity for Use in a Linear Collider

Donglin Zu and Jiaerh Chen

Abstract—**A nine-cell superconducting RF accelerating cavity is designed for the TeV electron linear accelerator collider in the next century. The ratio of the maximum surface electric field to the accelerating gradient, $E_{pk}/E_{\rm acc}$, is reduced to 2.024 and the cell-to-cell coupling remains as high as 1.95%. The distribution of the higher-order mode passbands is reasonable. There is no overlap between these bands, therefore no trapped modes. The circle-straight/line-ellipse-type structure provides good mechanical strength in the accelerating cavity. According to the present state of the art of surface processing techniques of Niobium cavities, it is possible to reach an accelerating gradient of 25–30 MV/m with beam load.**

Index Terms—**Accelerating gradient, linear collider, multicell Niobium cavity, SRF, trapped modes.**

I. INTRODUCTION

THE MAXIMUM beam energy accelerated in existing electron storage ring colliders is limited to under 0.1 TeV due to the synchrotron radiation loss, increasing in proportion to the fourth power of beam energy. A linear collider has no significant radiation loss, and its cost increases linearly with the beam energy. Therefore, to realize TeV range electron–positron collisions, a linear collider is needed. There is widespread consensus among the high-energy accelerator community that an e^+e^- collider with a center-of-mass energy of 2 × 0.5 TeV and luminosity of a few times 10^{33} $\text{cm}^{-2}\text{s}^{-1}$ should be the next accelerator [1] after the LHC at CERN. Such a collider would provide a powerful means for the top quark analysis via $t-\bar{t}$ production and also have the potential for the discovery of new particles.

The advantages of the TeV superconducting linear accelerator (TESLA) collider [2] is to allow low RF peak power (using relatively long pulse length) and average power (using multibunch). However, the key problem is that the accelerating gradient at present is not high enough, as it needs to have a substantial upgrade. The principle cost of the accelerating structure itself also needs to be reduced further. At the anticipated capital cost of the linear collider, for a superconducting RF(SRF) accelerating structure to prevail over a normal RF(NRF) accelerating structure, reliable operating gradients of the SRF cavity must reach 20 MV/m or above. Conventional accelerators have used 5 MV/m for a reliable operating gradient with beam load. The cost of an SRF cavity typically is 200 or 40 k$/MV. CEBAF uses 360 five-cell cavities. Suppose the accelerating gradient reaches 25 MV/m, the active length of 2 × 0.5 TeV linear collider will reach 2 × 20 km. The active length of a 1.3 GHz, nine-cell SRF cavity is about 1 m long. This TESLA collider requires 2 × 20 000 cavities of this kind. This paper studies the optimum shape design of the TESLA cavity.

The accelerating gradient of SRF cavities at present is limited by two phenomena, i.e., field emission (FE) and thermal breakdown [3]. The maximum electric field of the internal surface in a Niobium cavity, E_{pk}, is limited to a certain value by FE. The E_{pk} threshold can be upgraded to 100 MV/m through high RF peak power processing [4]. Even when producing a batch of SRF cavities using a Niobium sheet from the same furnace, using the same processing protocol, under the same machine conditions, their E_{pk}'s show variation. Despite using the same RF peak power processing, it is impossible for all the E_{pk} to reach 100 MV/m. It is likely for their low limit to be 75 MV/m. As the gradient without load depends on the $E_{pk}/E_{\rm acc}$ ratio, suppose a good cavity shape with a low $E_{pk}/E_{\rm acc}$ of 2.0 can be found, then the accelerating gradient without beam load could reach 37.5 MV/m. The accelerating gradient with beam load becomes two-thirds of its gradient without beam load, thus the gradient with beam load can reach 25 MV/m. It is difficult to make $E_{pk}/E_{\rm acc} < 2.0$ as $E_{pk}/E_{\rm acc}$ is restricted strongly by cell-to-cell coupling k. A compromise $E_{pk}/E_{\rm acc} \rightarrow 2, k > 1.8\%$ is needed and likely.

Thermal breakdown demands that the ratio of the maximum surface magnetic field to the accelerating gradient $H_{pk}/E_{\rm acc}$ reaches a minimum value. But this parameter has more room and is not a principal limit factor. In general, $E_{pk}/E_{\rm acc}$, k, and $H_{pk}/E_{\rm acc}$ all depend on the cavity cell shape, among them the FE is the main limiting factor.

II. THE CONCEPT AND THEORY OF MULTICELL ACCELERATING CAVITIES

The cells in a multicell cavity behave like weak-coupling oscillators whether there are standing wave modes or traveling wave modes. One mode of a single-cell cavity could split into N modes of a multicell cavity when the single-cell cavity is evolved into the N-cell cavity. The N modes possess slightly different frequencies which form a “passband” with a different phase shift in each cell. In the same passband, if the eigen circular frequency of the qth mode is ω_q and the longitudinal electric field in the nth cell is $En(q,\, t)$, their relation can be

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TABLE I
COMPARISON OF SRF ACCELERATING CAVITY FOR NEXT LINEAR COLLIDER

	LEP*[7]	Cornell[8]	KEK1[7]	KEK2[10]	Beijing[12]	Saclay[11]	DESY[11]
N_{cell} per cavity	4	10	9	9	9	9	9
OR(mm)	102.20	109.00	104.15	103.30	103.60	102.20	103.30
IR(mm)	32.67	40.90	40.00	38.00	35.80	32.31	35.00
N_{mesh}	20000	20000	20000	25000	25000	25000	25000
H_{pk}/E_{acc}((A/m)/(MV/m))	3129.8	4321.1	3967.7	3429.8	3414.7	3159.2	3318.4
k(%)	2.00	1.80	1.22	2.66	1.95	1.42	1.85
R/Q() per cell	121.48	90.96	112.92	108.00	110.87	124.33	115.22
E_{pk}/E_{acc}	2.359	2.054	2.039	2.220	2.024	2.000	2.070
cell type	CSC	CSC	SCSCS	CSE	CSE	CSC	CSE
trapped modes	no			no	no		

* LEP is not TESLA cavity, introduced as a reference cavity.

expressed with the following formulas:

$$\omega_q = \omega_0[1 + k(1 - \cos\Phi_q)]^{1/2} \quad (q = 1, 2, \cdots, N) \tag{1}$$

$$E_n(q, t) = E_0 \sin\left(\frac{2n-1}{2}\Phi_q\right)\cos\omega_q t \quad (n, q = 1, 2, \cdots, N). \tag{2}$$

Here k is the cell-to-cell coupling, E_0 is the maximum of the longitudinal electric field, and Φ_q is the phase shift in the qth cell.

$$\Phi_q = q \cdot \frac{\pi}{N} \qquad (q = 1, 2, \cdots, N) \tag{3}$$

when $q = N$, $\Phi_q = \Phi_N = \pi$, e.g., the oscillation phase difference in adjacent cells is π, and it is called a π mode. The π mode in the fundamental TM010 band is the accelerating mode to be used. When $q = 1$ and $\Phi_q = \Phi_1 = \pi/N$ in adjacent cells, the oscillation has a phase difference of π/N. When $N \rightarrow \infty$, $\Phi_1 \rightarrow 0$, so it is called "zero mode." Tuning is to make each cell resonate at the same frequency ω_N through mechanical adjustment and to make the peak value of $E_n(N, t)$ in each cell is equal. Cell-to-cell coupling k can be expressed as

$$k = \frac{f_\pi^2 - f_{\pi/N}^2}{2f_{\pi/N}^2 - [1 - \cos(\pi/N)]f_\pi^2}. \tag{4}$$

Here f_π is the mode frequency and $f_{\pi/N}$ is the "zero mode" frequency. The k tends to go down with the increasing cell, such as when an existing four-cell or five-cell cavity is evolved into nine-cell cavity, and its k value will become intolerably small. A more serious issue is that as the mode density in each passband increases, the accompanying number of cells increases, and even passband overlapping appears, thereby resulting in "trapped modes" which will destroy the normal working conditions of SRF accelerating cavity. The trapped modes were analyzed with a computer [5]. The physical mechanisms of generating trapped modes are not clear yet. In the design of SRF cavities, trapped modes are a troublesome problem. So seeking a new cavity shape without trapped modes is an important task for the cavity designer. Another parameter of a multicell cavity $\Delta f/f$ represents the dispersion of passband, and its definition is

$$\frac{\Delta f}{f} = \frac{2(f_\pi - f_{\pi/N})}{f_\pi + f_{\pi/N}}. \tag{5}$$

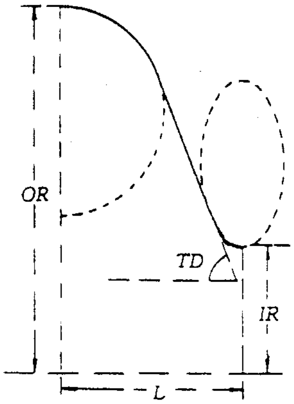

Fig. 1. BT cavity shape.

There is only one mode (π mode) useful for accelerating in the fundamental passband; the rest of the $N-1$ modes are harmful and need to be extracted by the main coupler. Hence the larger the dispersion of the fundamental passband, while the smaller the dispersion of higher order passbands, the better, so as to avoid passbands too wide to be separated. And it is desirable that passbands distribute homogeneously and reasonably.

III. SEARCHING FOR A NEW CAVITY SHAPE

To reduce the construction cost of the future linear collider and considering both the operating experience of SRF accelerating cavities at active service and the present state of the art of the surface processing techniques of SRF cavities, the new accelerating structure should satisfy the following conditions.

1) The number of cells per cavity should double to nine cells in order to reduce the number of expensive RF couplers, to save space, and to shorten the total length of the collider.
2) In the fundamental passband, cell-to-cell coupling k must be larger than 1.8%, and the higher, the better.
3) E_{pk}/E_{acc} must be reduced to 2.0 to make the accelerating gradient as high as possible.
4) The higher order modes (HOM) coupler mounted on the beam tube near the cell can efficiently extract the

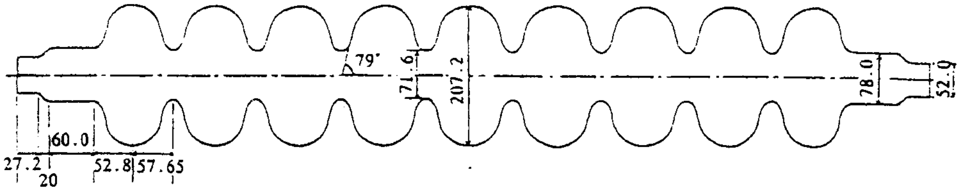

Fig. 2. The geometric structure of SRF nine-cell accelerating cavity for linear collider; the dimensions correspond to 1.3 GHz.

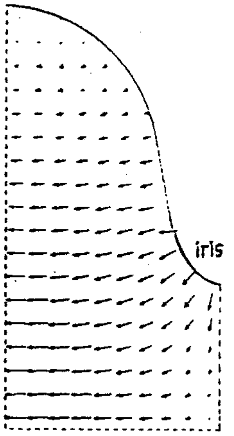

Fig. 3. The electric field distribution of π mode of TM010 band in the BT cell. The arrow indicates the field direction, and the length of arrow is proportional to field strength.

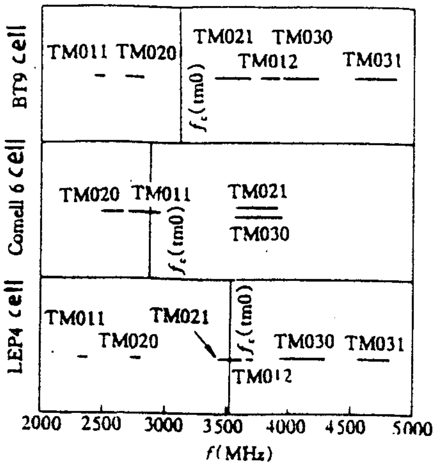

Fig. 4. The monopole HOM passbands distribution of the BT nine-cell cavity, LEP four-cell cavity, and Cornell six-cell cavity. The cutoff frequency of TM0-modes (tm0); f_c is the boundary between standing and travel modes.

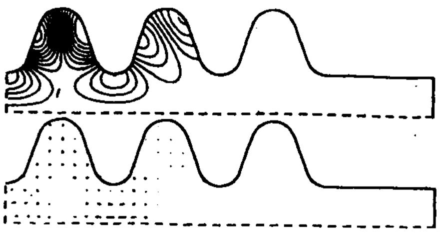

Fig. 5. There appear trapped modes in the Cornell six-cell cavity. The electromagnetic field in end cell nearly equals zero, and the cell's energy cannot propagate out of the cavity.

energy of higher-order standing-wave modes to make their external $Q_{\rm ext}$ low enough to be a tolerable level.

5) All traveling HOM's can propagate out of cells; so-called trapped modes are not allowed in cavities.
6) The structure should be reasonable, strong enough, and mechanically tunable.

The structure parameters of a cavity are usually described using one-fourth of the lengthwise section of an SRF cavity with axial symmetry, where the cell is the elemental block of a cavity. Half a cell wall consists of several segments of curves, such as circle-straight/line-circle (CSC) or ellipse-straight/line-ellipse (ESE). The "CSC" type has five independent parameters [6], i.e., cell outer radius OR, beam tube radius IR, iris radius R1, wall slope TD, and half-cell length L, respectively. First among them, L depends on the accelerating mode frequency because accelerated electrons must be synchronous with RF oscillation. In consideration of mechanical stability, we take TD = 79°, thus only the other three parameters, i.e., OR, IR, and R1 are allowed to vary. Our strategy is to keep right frequency 1.3 GHz resonant to keep a certain OR through adjusting IR and R1, due to each parameter variation that affects π mode frequency of the fundamental passband.

If 352-MHz, 500-MHz, and 1.5-GHz cavities are all scaled to 1.3 GHz, the OR, IR, and R1 of the LEP four-cell cavity [7] are the minimum (there are no trapped modes in it); in opposition the OR, IR, and R1 of Cornell ten-cell TESLA cavity [8] are the maximum (there appear trapped modes in six-cell cavity of this TESLA shape). We tried to find a good shape in which OR, IR, and R1 were between the two sets of extreme parameters. In the searching process, it is important to keep the resonant frequency of the trial cavity at 1.3 GHz. Otherwise, the $E_{pk}/E_{\rm acc}$ calculated will be incorrect. In addition, $E_{pk}/E_{\rm acc}$ will be incorrect [6] as well if the number of mesh points used by URMEL [9] is less than 25 000.

Using the "CSC" type, although searching for quite some time, we failed to reach the goal. We had to try a new shape, circle-straight/line-ellipse (CSE). At last we found a cell shape shown in Fig. 1, named the Beijing Tesla (BT) shape. The $E_{pk}/E_{\rm acc} = 2.024$, $k = 1.95\%$ for the accelerating mode of the nine-cell accelerating cavity (shown in Fig. 2) increased with this new cell shape.

IV. THE CHARACTERISTICS OF THE ACCELERATING MODE OF THE BT CELL

In order to command the accuracy and reliability of calculating the BT cell, we repeated the calculation for the Cornell TESLA, KEK TESLA, and non-TESLA LEP four-cell cavities, and the results agree with the data published

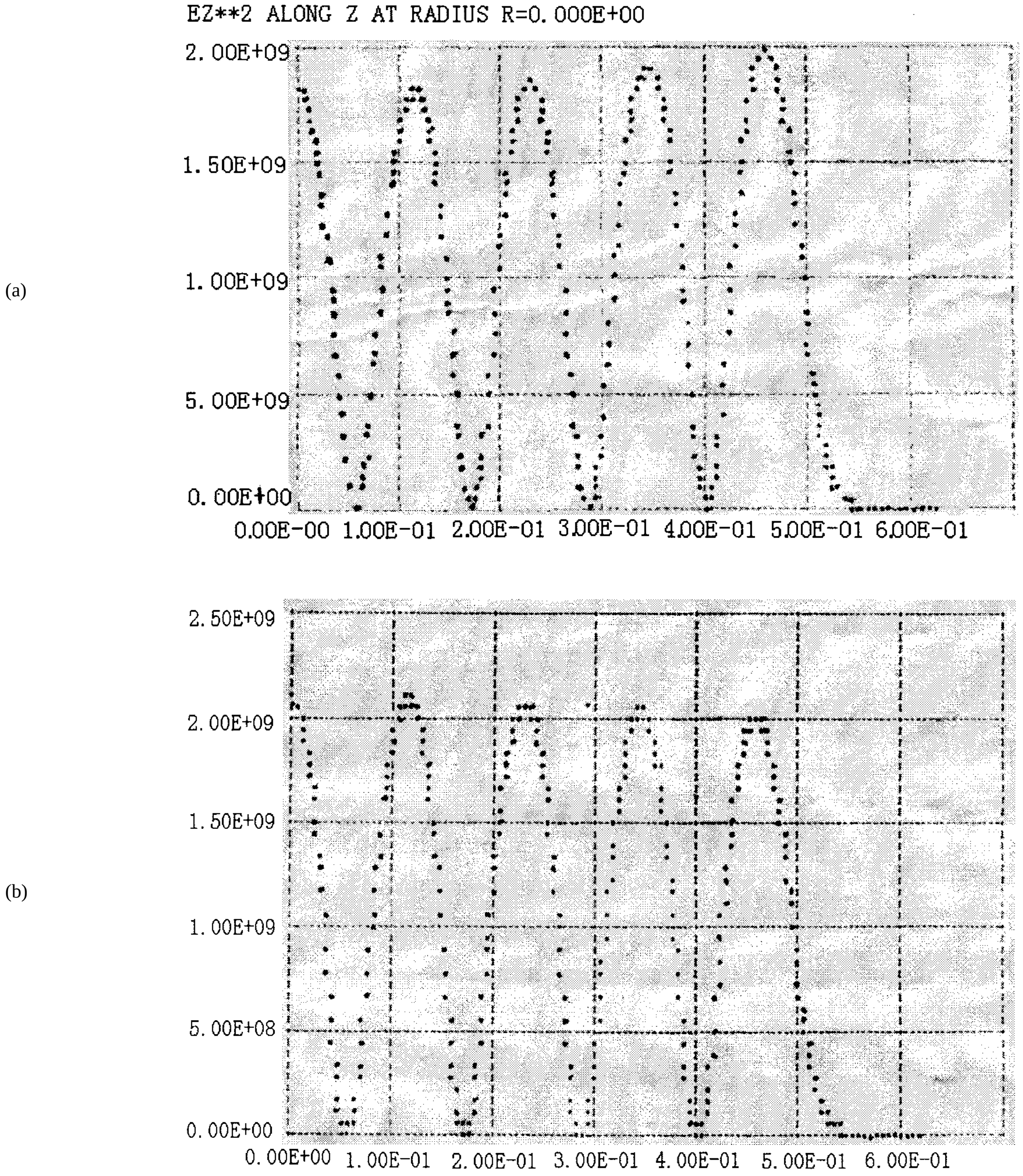

Fig. 6. The computer tuning of nine-cell BT structure with URMEL: (a) tuning curve when $L = 52.7$ mm and (b) tuning curve when $L = 52.8$ mm.

[8]. For comparison, Table I lists their cell number per cavity, two geometric parameters, the number of mesh points used in running URMEL, four physical parameters, cell type, and trapped modes. Among them there are two key parameters: $E_{pk}/E_{\rm acc}$ and k. The smaller the $E_{pk}/E_{\rm acc}$, the higher the accelerating gradient will be obtained; the larger the k value, the easier the adjusting, the tuning, and the controlling of the cavities are. However, $E_{pk}/E_{\rm acc}$ and k condition each other strongly. Generally speaking, to keep k larger than 1.8%, one cannot lower the $E_{pk}/E_{\rm acc}$ at will.

V. MONOPOLE HOM PASSBANDS AND THE TRAPPED MODES PROBLEM

In 1989, Cornell University was the first to design a ten-cell TESLA shape cavity [8], "CSC" type, with $E_{pk}/E_{\rm acc} = 2.10$, $k = 1.8\%$. It is necessary to reduce the ratio $E_{pk}/E_{\rm acc}$ through improving the geometry of cell structure so as to raise $E_{\rm acc}$ as high as possible, because raising E_{pk} threshold experimentally is restricted. Usually the maximum E_{pk} appears in the iris area where the electric field lines concentrate, as shown in Fig. 3. If the circle radius R1 at the iris is enlarged, the electric field lines will be diverged out; consequently, the maximum E_{pk} will go down. This is indeed an effective way for the accelerating mode. Cornell/CEBAF cavity's $E_{pk}/E_{\rm acc} = 2.56$, but the TESLA cavity $E_{pk}/E_{\rm acc}$ is reduced to 2.10. However, this way results in some difficulty for cell-to-cell coupling and propagation of HOM's. In the six-cell cavity, with this cell shape, under three times the fundamental frequency there already appears to be an overlap of TM021 and TM030 passbands (see Fig. 4), and thereby the probability of trapped modes is formed. This convinced us to change the circle at the iris to an ellipse so that the ellipse at the iris not only diverges the electric field lines and lowers the maximum E_{pk}, but also compresses the physical width of the iris area and facilitates the stagger of HOM passbands.

The "CSE"-type structure is advantageous to cell-to-cell coupling and propagation of HOM's. On the one hand it lowers E_{pk}/E_{acc}. On the other hand it raises cell-to-cell coupling k, as to avoid trapped modes formed. Another disadvantage of the Cornell TESLA is that TD $= 70°$ is not firm enough for mechanical stability. Its one-cell cavity collapsed once when it was pumped down. The mechanical stability of both the ESE type of Cornell/CEBAF five-cell cavity and CSC type of LEP four-cell is all verified to be strong enough. Taking LEP four-cell cavity's TD $= 79.09°$ as a good reference, in the Beijing CSE-type cavity we have a TD $= 79°$. It is believed that mechanical stability of CSE type is good enough. After the CSE cell type was chosen, searching for a good cell shape carefully in a wide range of geometry parameters was conducted, and the BT cell shape was obtained at last. Its HOM passbands distribution is reasonable compared with Cornell TESLA six-cell's and LEP four-cell's. In the Cornell six-cell there appears an overlap of HOM traveling wave modes; frequency degeneration modes from different bands; interaction, drawing each other and energy exchange between modes with the same or near frequencies; and consequently, a result in interference effect. The electromagnetic field enhances in a local area; meanwhile, the electromagnetic field reduces and even eliminates another local area, thereby the normal cell-to-cell coupling and propagation characteristics are changed. When the electromagnetic field of this kind of interference modes in end-cells becomes zero (shown in Fig. 5), the energy cannot propagate out of the cavities into beam tubes. This kind of "propagating" mode stays in a cavity as if in a trap, and thus they are named "trapped modes." A DESY TESLA cavity in design process does not exclude the possibility of trapped modes existing. A BT nine-cell cavity, like the LEP four-cell cavity shown in Fig. 4, has no overlapping passbands of monopole mode under 3.7 times the fundamental frequency. All mode frequencies are discrete. There is no condition of interference, therefore there are no trapped modes existing in it.

VI. COMPUTER TUNING

In a multicell cavity, to guarantee the end-cell and the in-cell having the same resonance frequency, the geometric dimensions of the half end-cell must be different from the in-cell because the beam tube effect needs to be compensated. We tuned the nine-cell BT cavity for the mode of the fundamental band by changing the end half cell length with URMEL, 25 000 mesh points. The half in-cell length equals 57.65 mm, the half end-cell lengths are 52.7 and 52.8 mm, respectively, corresponding to two sets of tuning curves indicated in Fig. 6(a) and (b). The field flatness ($\Delta E_z/\overline{E}_z$) is nearly 0.028. The tuning sensitivity is 2880 Hz per micrometer.

In the end cell, a slight mistune to HOM's is possible but not a concern. HOM's always manage to be damped adequately. The higher standing wave modes spread all over the cavity through cell-to-cell coupling and finally are extracted by the HOM coupler. The normal higher travel wave modes can always propagate through cells to the beam tubes in spite of mistune or not in the end cells as long as the trapped modes do not exist.

VII. CONCLUSION

A BT nine-cell cavity [12] for the next linear collider has no trapped modes, its E_{pk}/E_{acc} is reduced to 2.024, and k is kept as high as 1.95%. Making use of RF peak power processing [4], as long as the FE threshold is raised to 75 MV/m or above, the accelerating gradient with empty load can reach 37.5 MV/m, corresponding to the gradient with beam load of 25 MV/m. If the FE threshold reaches 90 MV/m, the accelerating gradient with beam load can reach 30 MV/m. Moreover, the CSE type is reasonable as well as the yield intensity being stable enough. In a word, the BT shape is an ideal and promising TESLA accelerating cavity candidate for the next linear collider.

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