Analysis Method for the Mooring System of a Flexible Floating Body

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Abstract:

The theory of three-dimensional hydroelasticity for free flexible floating bodies and the theory of mooring system analysis are integrated into a complete theoretical framework for analyzing the dynamical response of the mooring system of a flexible floating body. The effect of the elastic deformations of the floating body on the mooring system analysis is investigated. The mathematical model based on the perturbation method for the dynamic tension response analysis of the mooring system is presented. Three-dimensional hydroelasticity theory and Goodman-Lance method are used to solve the hydroelastic response of the floating body and dynamic tension response of the mooring line respectively. Their coupling relationship are also given and used to solve motion equations of the mooring lines. The theory presented in the paper is illustrated by a numerical example. And the results show that the elasticity of the very large floating structure has considerable effect on the dynamic responses of the mooring system.

1. Introduction

Hydroelasticity theory of floating bodies, which embody the full complexities of the dynamics of the structure concerned and the fluid around it, provides a more consistent and more rational approach for the assessment of the overall behavior of a flexible marine structure in waves. The theory has been developed for more than 20 years and in various forms such as two-dimensional theory, three-dimensional theory and non-linear theory (Bishop and Price, 1979; Wu, 1984; Wu et al., 1997; Chen, 2001;Chen et al.,2003b). Hydroelasticity theory has been used in many fields especially in the dynamic response analysis of very large floating structures (Ertekin et al., 1991; Watanabe et al., 1996; Ertekin et. al., 1999; Song et.al., 2002). However, few reports have dealt with the mooring systems. With the rigid assumption of the floating body, Huang et al. (2001) developed a method of perturbation up to second order in frequency domain to calculate the motion and the load of a moored floating structure in waves. Considering the dynamic interactions, they presented the slowly

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varying response of the structure and the dynamic characteristics of the mooring lines. The numerical results are validated through comparison with experiments. Ignoring the dynamic effect of the mooring lines, Chen et al. (2003b) presented the coefficients of the restoring forces of a mooring system acting on a flexible floating body. They presented the linear and nonlinear three-dimensional hydroelastic motion equations of a moored floating body in frequency domain, and a moored floating beam is presented as a numerical example. A hybrid method of frequency domain and time domain was developed by Chen et al. (2001) to predict the motion responses of a flexibly joint multi-body floating system in irregular waves. In this paper, the mathematical model based on perturbation method for the dynamic response analysis of a moored flexible floating body considering the dynamic effect of the mooring lines is presented, and the expressions of the coupling relationship between the motion of the floating structure and the mooring lines are also presented.

2. The Motion Equations of a Moored Flexible Floating Body

The motion equations of a moored flexible floating body can be expressed as (Chen et al. 2001)

$$[a+A]\{\ddot{p}\}+[b+B]\{\dot{p}\}+[c+C]\{p\}=\{F_w\}+\{F_m\}+\{R\}+\{Q\}, \tag{1}$$

where $[a]$, $[b]$ and $[c]$ are the generalised mass matrix, the generalised damping matrix and the generalised stiffness matrix of structure respectively. $[A]$, $[B]$ and $[C]$ are the generalised added mass matrix, the generalised added damping matrix and the generalised restoring matrix respectively. $\{p\}$ is the generalised principal coordinate vector. Furthermore, $\{F_w\}$, $\{F_m\}$, $\{R\}$ and $\{Q\}$ are the generalised wave exciting forces, the generalised mooring forces, the generalised static forces and the generalised gravity forces. For the floating body with uniformly mass distribution, the generalised static forces are equilibrant with the generalised gravity forces. Then the liner motion equations of a moored flexible floating body in frequency domain can be expressed as

$$\left[-\omega^2[a+A]+i\omega[b+B]+[c+C]\right]\{\bar{p}_I\}=\{\bar{F}_{wI}\}+\{\bar{F}_{mI}\}, \tag{2}$$

where ω is the incident wave circular frequency, $\{\bar{p}_1\}$ is the first order generalized principal coordinate vector. $\{\bar{F}_{w1}\}$ and $\{\bar{F}_{m1}\}$ are the first order generalized wave exciting forces and the first order generalized mooring forces. $\{\bar{F}_{m1}\}$ can be expressed as

$$\{\bar{F}_{m1}\}=\sum_{t=1}^{n}[D]^T\{\bar{f}_1\}_t, \tag{3}$$

where n is the number of the mooring line, $[D]$ is the mode vector matrix including the rigid and elastic modes, $\{\bar{f}_1\}_i$ is the first order forces vector acting on the floating body by the i-th mooring line, and $\{\bar{f}_1\}_i$ can be expressed as

$$\{\bar{f}_1\}_i = \{\{0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\} \quad \dots \quad \{\bar{X}_{1i} \quad \bar{Y}_{1i} \quad \bar{Z}_{1i} \quad 0 \quad 0 \quad 0\}_j \quad \dots\}^T, \qquad (4)$$

where X_{1i}, Y_{1i} and Z_{1i} are the first order forces acting on the floating body in X, Y, Z axis direction respectively induced by the i-th mooring line. In order to simplify the expressions, two coordinate systems are introduced, namely the floating body frame of $Oxyz$, and the coordinate system for mooring line, as shown in Fig.1.

According to the geometry relationship, one obtains the forces vector acting on the floating body induced by the i-th mooring line (Fan et al., 1998)

$$\begin{cases} X_i = -T_i \cos\varphi_i \cos\theta_i \\ Y_i = T_i \cos\varphi_i \sin\theta_i \\ Z_i = T_i \sin\varphi_i \end{cases}, \qquad (5)$$

where T_i, φ_i and θ_i denote the amplitudes of the tension of the i-th mooring line, the inclination (angle between the horizontal and the tangent directions of the mooring line) and the angle between the vertical plane mooring line located and the vertical plane including x axis, as shown in Fig.1.

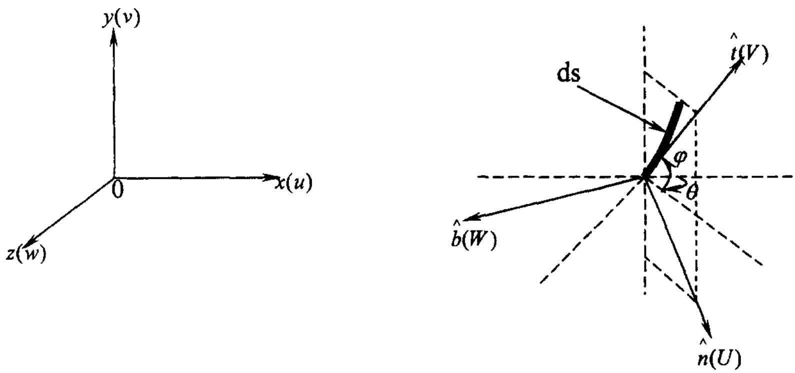


Fig.1 Coordinate system for mooring line

For dynamic analysis of the mooring line perturbing up to the second order, according to equations (5), the first order dynamic forces acting on the floating body induced by the i-th mooring line can be expressed as (Fan et al., 1998)

$$\begin{cases} \overline{X}_{1i} = T_{0i}\sin\varphi_{0i}\cos\theta_{0i}\overline{\varphi}_{1i} + T_{0i}\cos\varphi_{0i}\cos\theta_{0i}\overline{\theta}_{1i} - \overline{T}_{1i}\cos\varphi_{0i}\cos\theta_{0i} \\ \overline{Y}_{1i} = -T_{0i}\sin\varphi_{0i}\sin\theta_{0i}\overline{\varphi}_{1i} + T_{0i}\cos\varphi_{0i}\cos\theta_{0i}\overline{\theta}_{1i} + \overline{T}_{1i}\cos\varphi_{0i}\sin\theta_{0i} \\ \overline{Z}_{1i} = -\overline{T}_{1i}\sin\varphi_{0i} - T_{0i}\cos\varphi_{0i}\overline{\varphi}_{1i} \end{cases}, \tag{6}$$

where T_{0i}, φ_{0i} , θ_{0i} and $\overline{T}_{1i}, \overline{\varphi}_{1i}$, $\overline{\theta}_{1i}$ represent the static and the first order perturbation components of the i-th mooring line respectively.

The principle coordinate responses of the equations (2) can be defined as

$$\{\overline{p}_1\} = \{\overline{p}_{w1}\} + \{\overline{p}_{m1}\}, \tag{7}$$

where $\{\overline{p}_{w1}\}$ and $\{\overline{p}_{m1}\}$ are the first order principle coordinate responses induced by the first order wave exciting forces and the first order dynamic mooring forces respectively(Fan et al., 1998). Furthermore, $\{\overline{p}_{m1}\}$ can be defined as

$$\{\overline{p}_{m1}\} = \sum_{i=1}^{n}\{p_{1\overline{T}_i}\overline{T}_{1i} + p_{1\overline{\varphi}i}\overline{\varphi}_{1i} + p_{1\overline{\theta}_i}\overline{\theta}_{1i}\}, \tag{8}$$

where $\{\overline{p}_{1\overline{T}_i}\}$, $\{\overline{p}_{1\overline{\varphi}i}\}$ and $\{\overline{p}_{1\overline{\theta}_i}\}$ are the first order principle coordinate responses induced by the first order dynamic tension $\overline{T}_{1i}$, the inclination $\overline{\varphi}_{1i}$ and the angle $\overline{\theta}_{1i}$ respectively. Substituting equations (8) into (7), the principle coordinate responses of the equations (2) can be rewritten as

$$\{\overline{p}_1\} = \{\overline{p}_{w1}\} + \sum_{i=1}^{n}\{p_{1\overline{T}_i}\overline{T}_{1i} + p_{1\overline{\varphi}i}\overline{\varphi}_{1i} + p_{1\overline{\theta}_i}\overline{\theta}_{1i}\}. \tag{9}$$

Then, the first order perturbation displacements induced by wave exciting forces and the mooring forces of one mooring point of the floating body can be obtained by

$$\{\overline{U}_1\} = [D]\{\overline{p}_1\}. \tag{10}$$

3. The Motion Equations of the Mooring Line

From the 6 motion equations of the mooring line, perturbing up to the second order, one obtains the first order motion equations of the mooring line in frequency domain (Huang et al., 2001; Fan et al., 1998)

$$\left[\frac{d\overline{\varphi}_1}{ds} \quad \frac{d\overline{T}_1}{ds} \quad \frac{d\overline{U}_1}{ds} \quad \frac{d\overline{V}_1}{ds} \quad \frac{d\overline{W}_1}{ds} \quad \frac{d\overline{\theta}_1}{ds}\right]^T = [B][\overline{\varphi}_1 \quad \overline{T}_1 \quad \overline{U}_1 \quad \overline{V}_1 \quad \overline{W}_1 \quad \overline{\theta}_1]^T, \tag{11}$$

with $[B]=$

$$\begin{bmatrix} \frac{F_\varphi(1+\varepsilon_0)-W_c\sin\varphi_0}{T_0} & \frac{-\varphi_{0s}+F_0/EA}{T_0} & \frac{-i\omega\mu+F_u(1+\varepsilon_0)}{T_0} & \frac{F_v(1+\varepsilon_0)}{T_0} & \frac{F_W(1+\varepsilon_0)}{T_0} & \frac{F_\theta(1+\varepsilon_0)}{T_0} \\ W_c\cos\varphi_0-G_\varphi(1+\varepsilon_0) & \frac{G_0}{EA} & -G_U(1+\varepsilon_0) & i\omega\mu-G_V(1+\varepsilon_0) & -G_W(1+\varepsilon_0) & -G_\theta(1+\varepsilon_0) \\ -i\omega(1+\varepsilon_0) & 0 & 0 & \varphi_{0s} & \sin\varphi_0\theta_{0s} & 0 \\ 0 & \frac{i\omega}{EA} & -\varphi_{0s} & 0 & \cos\varphi_0\theta_{0s} & 0 \\ 0 & 0 & -\sin\varphi_0\theta_{0s} & -\cos\varphi_0\theta_{0s} & 0 & i\omega(1+\varepsilon_0)\cos\varphi_0 \\ \frac{T_0\cos\varphi_0\theta_{0s}-H_\varphi(1+\varepsilon_0)}{T_0\cos\varphi_0} & \frac{H_0/EA-\cos\varphi_0\theta_{0s}}{T_0\cos\varphi_0} & \frac{-H_U(1+\varepsilon_0)}{T_0\cos\varphi_0} & \frac{-H_V(1+\varepsilon_0)}{T_0\cos\varphi_0} & \frac{-i\omega\mu-H_W(1+\varepsilon_0)}{T_0\cos\varphi_0} & \frac{-H_\theta(1+\varepsilon_0)}{T_0\cos\varphi_0} \end{bmatrix}, \tag{12}$$

where U, W and V are the normal velocity, the tangential velocity and the binomial velocity respectively, they are shown in Fig.1. ε and W_c are the strain and the unit length weight in water of the mooring line respectively. F, H and G are the hydrodynamic force of the unit length of the mooring line in $\hat{n}$, $\hat{b}$ and $\hat{t}$ direction respectively. E and A are the effective static elastic modulus and the section area of the mooring line respectively. The subscript s,U,U,W,φ and θ denote the partial derivatives with respect to these variables, and subscript 0 and 1 denote the static and first order perturbation variables.

The relationship between the mooring point's velocity in the coordinate system for mooring line and the velocity in the floating body frame (as shown in Fig.1) can be expressed as

$$\begin{Bmatrix} \overline{U}_{1l} \\ \overline{V}_{1l} \\ \overline{W}_{1l} \end{Bmatrix} = [L_{mp}] \begin{Bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{Bmatrix}, \tag{13}$$

with

$$[L_{mp}] = \begin{bmatrix} \cos\theta_0\sin\varphi_0 & -\cos\varphi_0 & \sin\theta_0\sin\varphi_0 \\ \cos\theta_0\cos\varphi_0 & \sin\varphi_0 & \sin\theta_0\cos\varphi_0 \\ \sin\theta_0 & 0 & \cos\theta_0 \end{bmatrix}.$$

The velocity in the floating body frame can be expressed as

$$\begin{Bmatrix} \overline{u} \\ \overline{v} \\ \overline{w} \end{Bmatrix} = i\omega[T][D_s]\{\overline{p}_1\}e^{i\omega t}, \tag{14}$$

where $[T]$ is the transformation matrix of the two coordinate systems. $[D_s]_{3*m} \in [D]$ is the displacement mode vector matrix corresponding to the mooring point. m is the mode number truncated to. Substituting equation (14) into equation (13), one obtains

$$\begin{Bmatrix} \overline{U}_{1i} \\ \overline{V}_{1i} \\ \overline{W}_{1i} \end{Bmatrix} = i\omega[L_{mp}][T][D_s]\{\overline{p}_1\}e^{i\omega t} = [LTD]\{\overline{p}_1\}e^{i\omega t}. \tag{15}$$

Substituting equation (9) into equation (15), one obtains the coupling relationship between the motion of the floating body and the mooring line

$$\begin{cases} \overline{U}_{1i} = \overline{U}_{1iw} + \sum_{j=1}^{n}(\overline{U}_{1i\overline{T}j}\overline{T}_{1j} + \overline{U}_{1i\overline{\varphi}j}\overline{\varphi}_{1j} + \overline{U}_{1i\overline{\theta}j}\overline{\theta}_{1j}) \\ \overline{V}_{1i} = \overline{V}_{1iw} + \sum_{j=1}^{n}(\overline{V}_{1i\overline{T}j}\overline{T}_{1j} + \overline{V}_{1i\overline{\varphi}j}\overline{\varphi}_{1j} + \overline{V}_{1i\overline{\theta}j}\overline{\theta}_{1j}) \\ \overline{W}_{1i} = \overline{W}_{1iw} + \sum_{i=1}^{n}(\overline{W}_{1i\overline{T}j}\overline{T}_{1j} + \overline{W}_{1i\overline{\varphi}j}\overline{\varphi}_{1j} + \overline{W}_{1i\overline{\theta}j}\overline{\theta}_{1j}) \end{cases}, \tag{16}$$

where $\overline{U}_{1iw}, \overline{V}_{1iw}$ and $\overline{W}_{1iw}$ are the first order velocity amplitudes of the mooring point in three directions induced by the i -th mooring line. ($\overline{U}_{1i\overline{T}j}$, $\overline{U}_{1i\overline{\varphi}j}$, $\overline{U}_{1i\overline{\theta}j}$), ($\overline{V}_{1i\overline{T}j}$, $\overline{V}_{1i\overline{\varphi}j}$, $\overline{V}_{1i\overline{\theta}j}$) and ($\overline{W}_{1i\overline{T}j}$, $\overline{W}_{1i\overline{\varphi}j}$, $\overline{W}_{1i\overline{\theta}j}$) are the fist order velocity amplitudes of the top point of the i-th mooring line in three directions induced by unit $\overline{T}$, unit $\overline{\varphi}$ and unit $\overline{\theta}$ of the j-th mooring line respectively.

The boundary condition of the bottom point of the i-th mooring line can be expressed as

$$\overline{U}_{1i} = \overline{V}_{1i} = \overline{W}_{1i} = 0. \tag{17}$$

With the whole boundary condition of equation (16) and equation (17), equation (11) can be solved by Goodman-Lance method (Huang et al., 2001; Fan, et. al., 1998). Then, one can obtains the variable $\overline{\varphi}_{1i}, \overline{T}_{1i}, \overline{V}_{1i}, \overline{U}_{1i}, \overline{W}_{1i}$ and $\overline{\theta}_{1i}$ at each top point of the mooring line.

4. Numerical Example

4.1. The model

A moored pontoon-type VLFS mentioned by Chen (Chen et al., 2003a) is chosen to illustrate the theory presented above. The particulars of the VLFS and the mooring system are shown respectively in Table 1 and Table 2, and the configuration of the mooring system is shown in Fig.2.

Table 1 Particulars of the VLFS

Length	L	300.0	m
Width	B	60.0	m
Depth	h	2.0	m
Draught	d	0.5	m
Bending rigidity	EI	4.77×10^{11}	Nm^2
Young's modulus	E	1.19×10^{10}	N/m^2
Poisson's ratio	ν	0.13	
Mass density	ρ	256.25	kg/m^3

Table 2 Particulars of the mooring system

Mooring Line No.	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8
Direction (degree)	0.0	45.0	90.0	135.0	180.0	225.0	270.0	315.0
Pretension(ton)	50.0	50.0	50.0	50.0	50.0	50.0	50.0	50.0

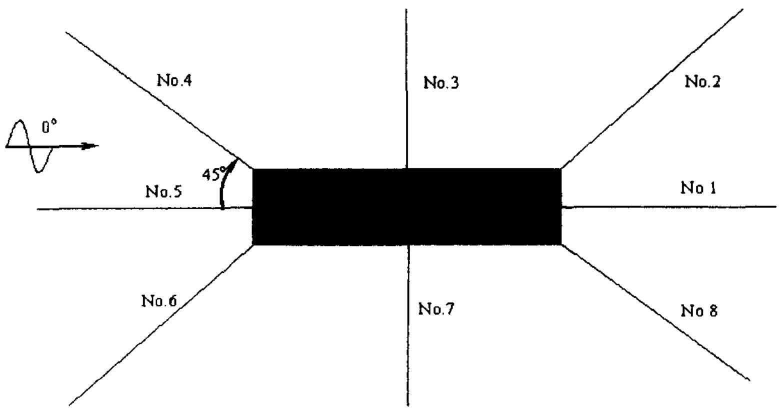
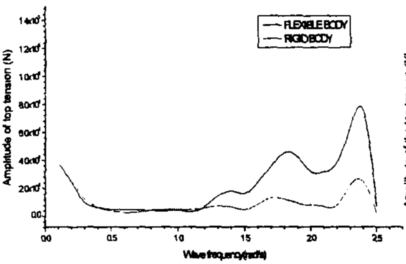


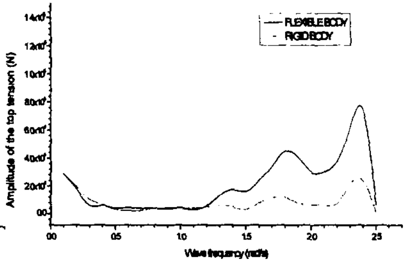
Fig.2 Configuration of the mooring system

4.2. Discussion of the mooring system of the flexible body

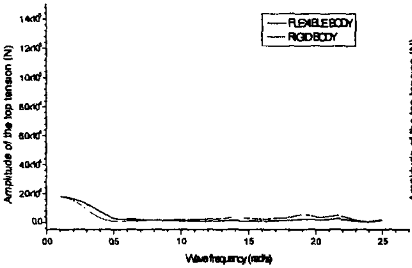
The results of the amplitudes of the dynamic tension of the mooring lines acting on the flexible and rigid floating body are shown in Fig.3. The incident wave angle and the wave amplitude are β=0°and ζ=1.0m respectively.



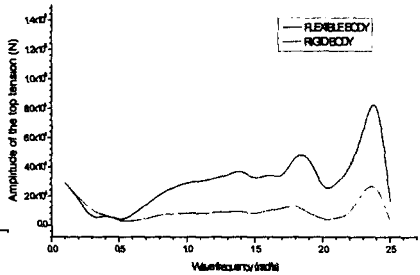
(a) Mooring line No.1



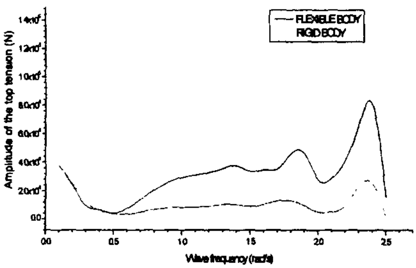
(b) Mooring line No.2



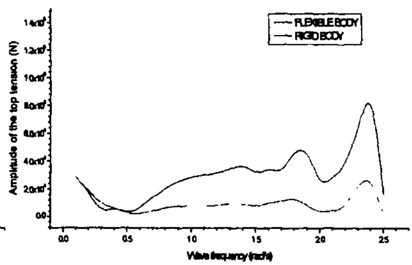
(c) Mooring line No.3



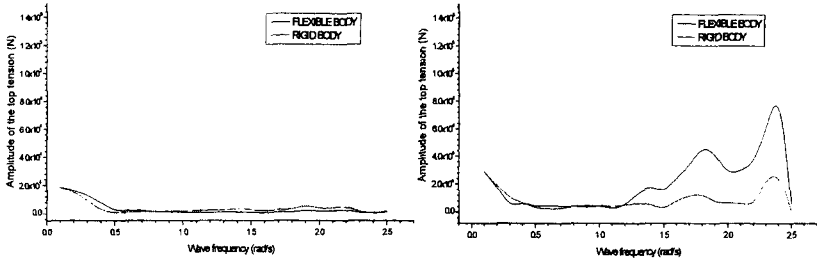
(d) Mooring line No.4



(e) Mooring line No.5



(f) Mooring line No.6



(g) Mooring line No.7 (h) Mooring line No.8

Fig.3 The numerical results of the dynamic amplitude of the mooring lines' top tension

Based on the curves shown in Fig.3, the frequency characteristics of the mooring lines' dynamic amplitude of top tension of a floating flexible body can be analyzed as follows.

a. Because of the symmetry of the structure of the floating body, the wave exciting forces and the positions of the mooring lines, the amplitudes of the top dynamic tensions of the mooring lines are also symmetrical.

b. At the middle part of the floating body, dynamic tensions of the mooring lines of the rigid body are almost equal to that of the flexible body, and approaches to zero with the frequency increasing.

c. At two ends of the floating body, the dynamic tensions of the mooring lines of the rigid body are almost equal to that of the flexible body at low wave frequency, and the latter is greater than the former at high wave frequency, which is sometimes 6 times greater.

5. Summany and Conclusions

The theory of three-dimensional hydroelasticity for unmoored floating bodies and the theory of mooring system analysis based on Goodman-Lance method have been integrated into a complete theoretical framework for analyzing the dynamic response of the mooring system of a flexible floating body. The effect of the elastic deformations of the floating body on the mooring system analysis has been studied. The numerical results have shown that the elastic deformations of the floating body cannot be negligible in the dynamic tension analysis of the mooring lines when the wave frequency is higher, but can be neglected when the wave frequency is lower.

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